Estimation de la probabilité de défaillance d'une éolienne marine (dans un contexte de conception)

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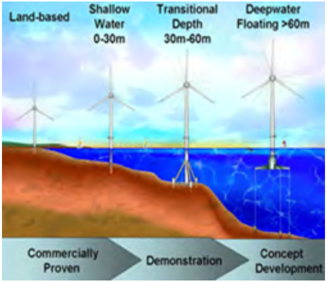
Offshore wind market : general context

- The offshore **wind market is growing** rapidly thanks to several drivers :
  - important wind resources
  - less turbulence in offshore wind
  - reduced use conflict than onshore wind
  - reduced visual impact

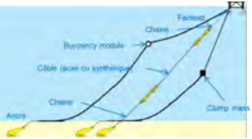
- The offshore wind market is looking at more distant and deeper locations for which **floating foundations are required**



source : NREL

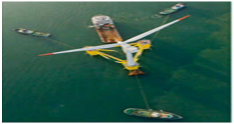
IFPEN's objective and approach

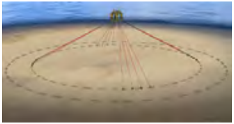
- The objective of IFPEN is to participate in the development of floating offshore wind turbines
  - by proposing reliable solutions with fit for purpose technologies to **lower the cost** of floating offshore wind
  - by offering a set of solutions for floater and **mooring technologies usable** for any standard offshore wind turbine



Design and reliability issues of a floating wind turbines

- Floating wind turbine design
  - a **key component** is the **anchoring system** which restrict the floating support motion.
  - anchoring **must avoid** a failure of the anchoring lines under **extreme stress** or/and **fatigue** during the lifespan of the structure
  - should have an **optimal cost** in order to be competitive
  - involve the use of an **expensive black-box simulator** (DeepLines^{TM}) modelling the complex system
    - given control strategy (pitch actuators, generator)
- Subjected to **random marine environment** : wind, wave





Stochastic model of wind/wave load :

- Subjected to **random marine environment** : wind, wave
  - different time scales : short and long term distinction
  - combined loading with different directions : numerous load cases to be simulated

- Stochastic model of wind/wave load :
  - **spectral model** : *stationary Gaussian process*, simulation via frequency spectrum (Kaimal, Jonswap), number of short-term parameters depends on simulation length($\geq 100 - 200$)
  - **Karhunen-Loève** : *generally non-Gaussian, non-stationary process*, distribution of parameters needs to be estimated (site dependent), number of parameters depends on desired variability

A reliable and competitive design

- **Design reliable against extremes**
  ⇒ Postdoctorate : A. Murangira (IFPEN)
  ⇒ Failure probability estimation

- Design reliable against fatigue
  ⇒ PhD Thesis : I. Aleksovska (IFPEN-P7)

- Design reliable against extremes and fatigue
  ⇒ Premature

Extreme case : an example of critical wind speed process and response

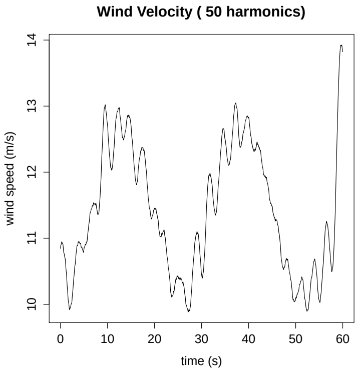


FIGURE : Critical wind (SQA)

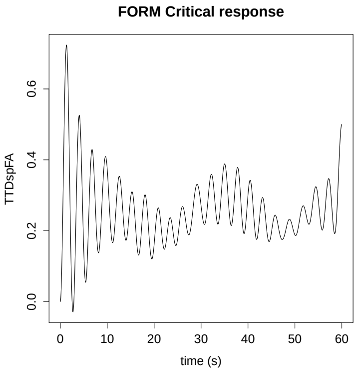


FIGURE : Critical response (SQA)

Design reliable against extremes

Find an **optimal anchor** design minimizing the costs under **feasibility and time dependent reliability to extreme chance constraints** :

$$\min_{\mathbf{x}_d} C(\mathbf{x}_d) = A\mathbf{x}_d,$$

$$\text{subject to} \begin{cases} \mathbf{h}(\mathbf{x}_d) \leq 0 \\ \mathbb{P}\big(\forall t \in [0,\, T], g(t, \mathbf{x}_d, \mathbf{X}(.), \mathbf{X}_{LT}) \leq s\big) \geq 1 - p \end{cases}$$

with

- $\mathbf{x}_d$ the design parameters
- $\mathbf{h}$ the feasibility constraints (good properties)
- g a black-box expensive simulator modelling the structure responses
- $\mathbf{X}(.)$ random processes modelling the environmental phenomenons
- $\mathbf{X}_{LT}$ random variables modelling the long term behaviour of the environmental processes
- s a limit threshold
- p a required level ($\sim 10^{-5}$)

Challenges

- **Complex phenomenon** to model (environmental conditions)

  $\Rightarrow$ Simulation chain is **CPU intensive** (DeepLines^{TM})

- **Probabilistic constraints**

  $\Rightarrow$ **Small** failure **probability** to estimate ($\leq 10^{-5}$) : standard Monte-Carlo impractical

  $\Rightarrow$ **Non-linear black-box failure function** g : makes the study of the admissible area more difficult

  $\Rightarrow$ **High dimensional** problem involved in the probabilistic constraint estimation : up to a few hundreds random variables for wind and wave time process models

- (**MINLP** optimization problem)

Reliability methodology. IFPEN Postdoctorate : A. Murangira

- methods based on design point alone yield an **estimate of failure probability with reasonable computational effort**
  - but without confidence intervals
  - accuracy highly dependent on non-linearities of mechanical output
- More **accurate estimation via Monte Carlo methods** (**high computing cost** for low P_f)
  - Importance sampling (with/without design point information)
  - Subset simulation,. . .
- may **reduce computational load via metamodels** (Kriging, SVM)
  - AK-MCS (Kriging + standard Monte Carlo) [Echard et. al. 2011]
  - Meta-IS (Kriging + Importance sampling) [Dubourg, 2011], . . .
- Most metamodels fail in high dimension

New approach : combine dimension reduction methods, Kriging and importance sampling

IS failure probability estimator

- $X \in \mathbb{R}^d$ input random vector with prior density q
- $\tilde{q}$ **proposal density** s.t. $\mathbb{1}_{g(x)\leq 0}\, q(x) \neq 0 \implies \tilde{q}(x) \neq 0$
- P_f estimation through IS : $x^{(i)} \sim_{i.i.d.} \tilde{q},\ i = 1, \dots, n$

$$\hat{P}_f^{IS} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{g(x^{(i)})\leq 0} \frac{q(x^{(i)})}{\tilde{q}(x^{(i)})} \tag{1}$$

- $\mathrm{Var}_{\tilde{q}}(\hat{P}_f^{IS}) = 0$ for the **optimal importance density** $\tilde{q}_{opt}(x) = \frac{\mathbb{1}_{g(x)\leq 0}\, q(x)}{P_f}$ but intractable sampling

Meta-IS : importance density

- Meta-IS [Dubourg, 2011] is based on the Kriging metamodel of the limit state function $g \sim \mathrm{GP}(\mathbf{f}^T\boldsymbol{\beta}, C)$
  - mean function $\mathbf{f}^T(\cdot)\boldsymbol{\beta}$ = linear regression model
  - stationary covariance function $C(x, x') = \sigma^2_{\tilde{G}} R_\theta(x - x')$

- $\mathbf{y} = [g(x^1), \dots, g(x^{N_D})]^T$: observation vector and $(\theta, \sigma^2_{\tilde{G}})$: GP parameters the **Kriging predictor** at a new data point x is

$$\hat{G}(x) \sim g(x) \mid \mathbf{y}, \theta, \sigma^2_{\tilde{G}} \sim \mathcal{N}(m_{\hat{G}}(x), \sigma^2_{\hat{G}}(x))$$

  where $m_{\hat{G}}$ and $\sigma_{\hat{G}}$ admit closed form expressions

- Obtain the *quasi-optimal importance density* $\tilde{q}_*$ by replacing $\mathbb{1}_{G(x)\leq 0}$ with the **probabilistic classification function**

$$\pi(x) = \mathbb{P}_0(\hat{G}(x) \leq 0) = \Phi\left(-\frac{m_{\hat{G}}(x)}{\sigma_{\hat{G}}(x)}\right) \quad (2)$$

$$\tilde{q}_*(x) = \frac{\pi(x)\, q(x)}{P_{f,\epsilon}} \quad (3)$$

Meta-IS : failure probability estimation

$$P_f^{Meta-IS} = \int \mathbb{1}_{g(x)\leq 0} \frac{q(x)}{\tilde{q}_*(x)} \tilde{q}_*(x) \mathrm{d}x = P_{f,\epsilon} \int \frac{\mathbb{1}_{g(x)\leq 0}}{\pi(x)} \tilde{q}_*(x) \mathrm{d}x \quad (4)$$

$$= P_{f,\epsilon} \alpha_{\mathrm{corr}} \quad (5)$$

- $P_{f,\epsilon} = \mathbb{E}_q(\pi(X))$ is the **augmented failure probability**, may be estimated by standard Monte Carlo simulation
- $\alpha_{\mathrm{corr}} = \mathbb{E}_{\tilde{q}_*}(\frac{\mathbb{1}_{g(X)\leq 0}}{\pi(X)})$ is a **correction factor** that can be estimated by using MCMC sampler targeting $\tilde{q}_*$
  $\alpha_{\mathrm{corr}} \to 1$ as Kriging model gets more accurate

Issues of metamodel-based importance sampling

- Kriging can break down in high dimension
- MCMC sampling in high dimension is more complex, although some efficient algorithms exist (modified Metropolis-Hastings [Au and Beck, 2001])
- Likewise, the estimation of $P_{f,\epsilon} = \mathbb{E}_q(\pi(X))$ is easier but still high-dimensional

Sufficient dimension reduction (SDR)

- X : input (vector of short term parameters)
  Y : output/response variable (limit state function output, failure domain indicator)
- **Objective** : find linear subspace of X predictors that contains all information on regression/classification of Y against X
- For $B \in \mathbb{R}^{d \times r}$, $S(B) = \mathrm{span}(B)$ is a *(sufficient) dimension reduction subspace* if

  $$Y \perp\!\!\!\perp X \mid B^T X$$

  or equivalently

  $$Y = g_r(B^T X, \varepsilon) \qquad \varepsilon \perp\!\!\!\perp X$$

  g_r : unknown link function
- **Estimation** : *Kernel dimension reduction* (KDR) [Fukumizu, 2008] or *gradient-KDR* [Fukumizu, 2014] almost no assumptions on distribution of X

Inferring the dimension in KDR

SDR model $Y = g_r(B^T X, \varepsilon)$, $\varepsilon \perp\!\!\!\perp X$, $B^T B = I_d$

- In standard KDR, $r = \mathrm{rank}(B)$ needs to be known in advance : we propose a **cross-validation** procedure to infer r
- **Kriging** : If $r << d$, metamodel of link function g_r obtainable in a manageable dimension
- no information on $\varepsilon \implies$ consider the simplified model

$$Y = g_r(B^T X) \tag{6}$$

$\hat{G}_r(z)$: Kriging predictor in SDR subspace

Meta-IS with sufficient dimension reduction

- We assume standard Gaussian inputs X
- Given a dimension reduction matrix B s.t. $B^T B = I_d$, we suggest the following quasi optimal IS density

$$\tilde{q}_*(x) = \frac{\pi_r(B^T x) q(x)}{P_{f,\epsilon}} \tag{7}$$

π_r : the probabilistic classification function of the Kriging metamodel in the SDR subspace

$$\pi_r(z) = \Phi\left(-\frac{m_{\hat{G}_r}(z)}{\sigma_{\hat{G}_r}(z)}\right)$$

- Metamodel update : let $z_{new} \in \mathbb{R}^r$ a point to be added to DoE.
  - In the simplified SDR model, $Y = g_r(B^T X)$, g_r is *unknown*
  - However $Y = g(X)$, g being the *known* limit state fun.
  - Hence, add $y_{new} = g((B^T)^{\#} z_{new}) = g_r(z_{new})$ to DoE where $(B^T)^{\#} B^T = I_d$

Meta-IS with sufficient dimension reduction : analysis in Gaussian case

Let $\tilde{q}_*(x) = \frac{\pi_r(B^T x) q(x)}{P_{f,\epsilon}}$ and $B_a = [B, B_\perp]$ where $B_\perp$ orthonormal basis of $\mathrm{span}(B)^\perp$

- **Lemma**

  (i) *The augmented failure probability $P_{f,\epsilon}$ can be expressed as $P_{f,\epsilon} = \mathbb{E}(\pi_r(Z))$ where Z is an r dimensional standard normal variable.*

  (ii) *Let $W_2 \sim \mathcal{N}(0_{d-r\times 1}, I_{d-r})$, $W_1 \sim \frac{\pi_r(w_1)\varphi_r(w_1)}{P_{f,\epsilon}}$ where φ_r is the standard r-dimensional normal pdf and $W = [W_1^T, W_2^T]^T$. Then $\tilde{X} = B_a^{-T} W$ is distributed according to $\tilde{q}_*(x)$.*

- **Conclusion** : estimation of $P_{f,\epsilon}$ is possible by sampling in $\mathbb{R}^r$. For the estimation of α_{corr}, sampling $\tilde{X} \sim \tilde{q}_*$ only requires MCMC sampling in $\mathbb{R}^r$ (instead of $\mathbb{R}^d$) and straightforward standard Gaussian generation in $\mathbb{R}^{d-r}$

The case of non-Gaussian inputs

- The reliability problem maybe reformulated in the Gaussian standard space $\mathbb{U}$ through Nataf transformation T for example : $U = T(X)$, $U \sim \mathcal{N}(0, I_d)$,

$$P_f = \mathbb{P}(g(T^{-1}(U)) \leq 0)$$

- quasi-optimal density

$$\tilde{q}_*(u) = \frac{\pi_r(B^T T^{-1}(u))\varphi_d(u)}{P_{f,\epsilon}}$$

- MCMC sampling now in full dimension $\mathbb{R}^d$, more complex but possible through modified Metropolis-Hastings [Au and Beck, 2001]

Academic example

limit state function $g(x) = 78 - \sum_{i=1}^{d} X_i - \sum_{i=1}^{3}(X_i + X_{i+1})^2$

- input $X = (X_1, \ldots, X_d)$, $d = 50$, X_i i.i.d. lognormal with mean 1, std 0.2
- $g(x) = 78 - a^T X - \|C^T X\|^2$, $a = (1 \cdots 1)^T$, C : rank 3 matrix $\Longrightarrow$ $\mathrm{span}([a, C])$ is DR subspace (dim 4)

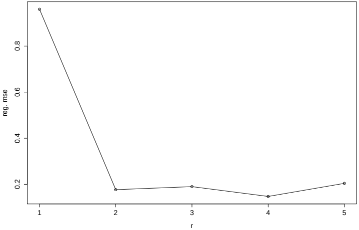


FIGURE : SDR dimension inference by CV

Method	FORM	Monte Carlo	IS-FORM	MetaIS	MetaIS-DR
N	306	1.5×10^6	5306	6551	2339
$\hat{P}_f$	4.58×10^{-5}	1.84×10^{-4}	1.87×10^{-4}	1.84×10^{-4}	1.82×10^{-4}
c.o.v.	-	6.0×10^{-2}	4.4×10^{-2}	5×10^{-2}	5×10^{-2}

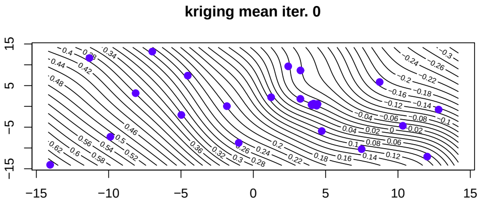
Failure probability of a WT submitted to wind load

Case study :

- Stationary conditions : mean wind speed/st.dev. $U_{10} = 11.5$ m/s
- Failure : $Y > 0.4$, Y : Tower top displacement (m)

- Preliminary **multiple design point** search using SQA with different starting points
- Implementation of **standard IS**
- Implementation of **Meta-IS with SDR** with estimated sufficient dimension $r = 2$
- Implementation of **subset simulation (SS)**

Metamodel refinement



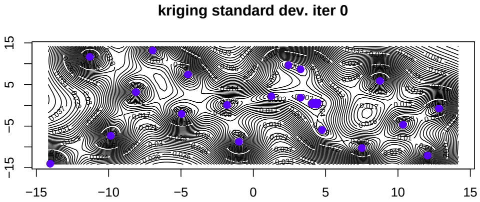


FIGURE : Kriging predictor

Metamodel refinement

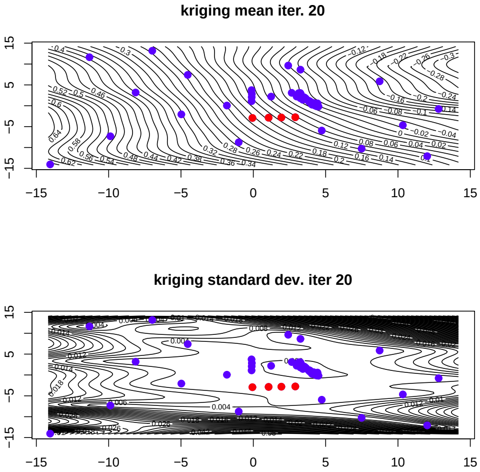


FIGURE : Kriging predictor

Metamodel refinement

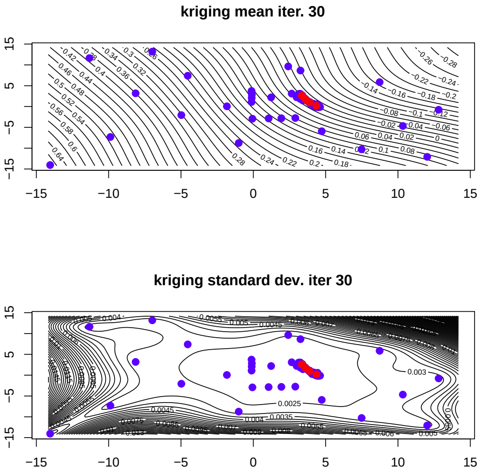


FIGURE : Kriging predictor

Metamodel refinement

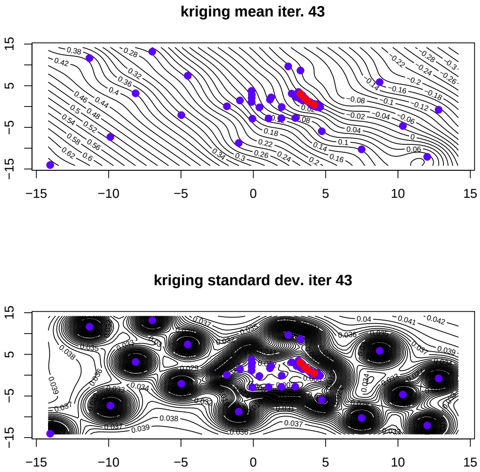


FIGURE : Kriging predictor

Simulation results

	$\hat{P}_f$	C.I.	# G calls	$P_{f,\epsilon}$	α_{corr}
Multi-FORM	3.78×10^{-6}*	N/A	4657	N/A	N/A
IS	2.68×10^{-5}	$[2.30, 3.05] \times 10^{-5}$	84657	N/A	N/A
Meta-IS-SDR	2.34×10^{-5}	$[2.07, 2.61] \times 10^{-5}$	10645	3.25×10^{-5}	0.7198
SS	2.73×10^{-5}	$[1.26, 4.20] \times 10^{-5}$	57000	N/A	N/A

TABLE : Failure probability estimation : method comparison

* : based on main design point

Conclusions and perspectives

- Meta-IS-SDR provides convenient dimension reduction framework for reliability analysis
- What if the inputs in the physical space aren't Gaussian ?
  - Dimension reduction in the physical space by subspace projection makes more sense in the physical space (e.g. if there are irrelevant inputs)
  - Could try SDR in transformed space $\mathbb{U}$ but lesser guarantee of validity of SDR model since transformed variables mix inputs
- We still need to assess the long term failure probability/incorporate wave elevation model
- We have carried out preliminary sensitivity analysis study to identify relevant inputs
  - in SDR, dimension reduction subspace not necessarily interpretable
  - Screening can be performed by using dependence measures (distance correlation, maximum mean discrepancy, etc.),...
  - ..., but these measures usually require a sample from $q(X \mid g(X) \leq 0)$
  - may be possible to target $q(X \mid g(X) \leq 0)$ using proposal density from Meta-IS-SDR ?